Parametric Activity 4

MATH 116-022 (Lutz)

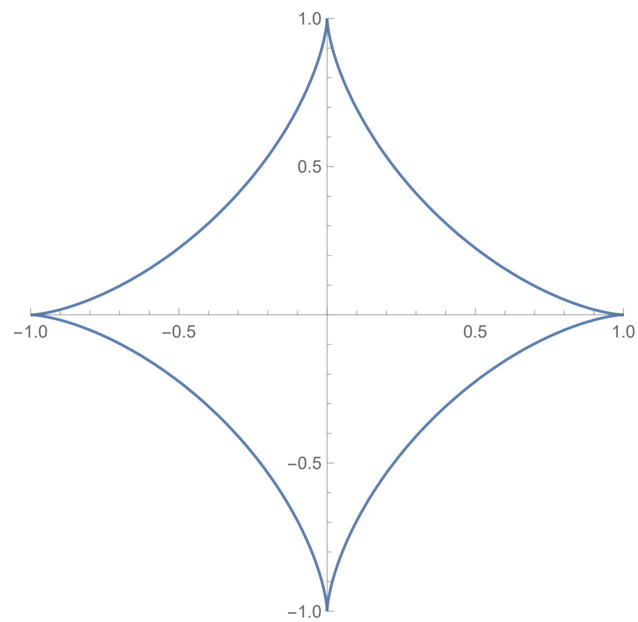
1. Pictured below is an astroid, the four-cusped member of the hypocycloid family. It is given by the parametric equations

$$x(t) = \cos^3(t)$$
$$y(t) = \sin^3(t).$$

   (a) Show that the parametrization above satisfies $x^{2/3} + y^{2/3} = 1$. (This is the Cartesian equation for the astroid, much like $x^2 + y^2 = 1$ for the unit circle.)

   (b) Compute the perimeter of the astroid by hand. (The identity $\sin(2t) = 2\cos(t)\sin(t)$ will almost certainly come in handy.)

   (c) What is the speed of the parametrization at $t = \frac{\pi}{4}$? (Remember: $\cos(\frac{\pi}{4}) = \sin(\frac{\pi}{4}) = \frac{\sqrt{2}}{2}$.)



2. The following are parametrizations of curves belonging to circles for $t \geq 0$. In each case, give (x, y)-coordinates for the beginning point, indicate which direction the parametrization is heading (clockwise or counterclockwise), and describe the **path** traced out by the parametrization.

   (a) $(x(t), y(t)) = (5\cos(t) + 6, 5\sin(t) + 8)$

   (b) $(x(t), y(t)) = (-\sin(t), \cos(t))$

   (c) $(x(t), y(t)) = (\cos(e^{-t}), \sin(e^{-t}))$

   (d) $(x(t), y(t)) = (\cos(\arctan(t)), -\sin(\arctan(t))$

   (e) $(x(t), y(t)) = (\cos(\sin(t)), \sin(\sin(t)))$

3. A particle's motion is parametrized by $(x(t), y(t))$ for $t \in [0, 5]$ where $x(t) = t^2$ and $y(t)$ is linear between the values given in the following table:

t	0	1	2	3	4	5
y(t)	3	0	2	-4	-6	3

(a) What is the speed of the particle at $t = 2.5$?

(b) Does it ever come to a stop?

(c) Sketch the curve; pay attention to concavity.

4. The Folium of Descartes, pictured below, has the parametrization

$$(x(t), y(t)) = \left(\frac{3t}{1+t^3}, \frac{3t^2}{1+t^3}\right).$$

The loopy part is the curve we actually want; the line underneath it is an unwanted asymptote!

(a) Show that the parametrization satisfies $x^3 + y^3 = 3xy$. (This is how Descartes presented the curve to Fermat, in an effort to test the latter's skills.)

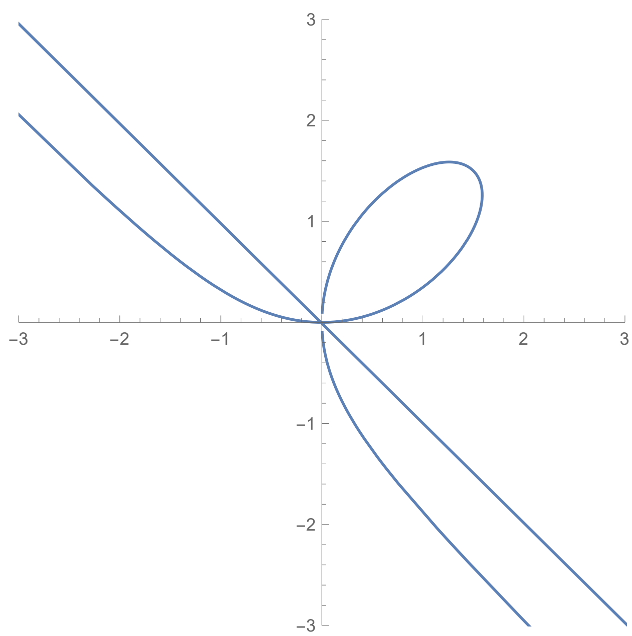
(b) When is the slope -1? This is the tip of the loop; give parametric and Cartesian equations for the tangent line. (Hint: $t^4 + 2t^3 - 2t - 1 = (t-1)(t+1)^3$.)

(c) What happens to the curve as $t \to -1^+$? What about $t \to -1^-$?

(d) Show that

$$x(t) + y(t) + 1 = \frac{(1+t)^2}{1-t+t^2}.$$

(e) Compute $\lim_{t\to-1} x(t) + y(t) + 1$. Use this to find the equation of the asymptote.



5. The following questions are (important) miscellany.

   (a) Give parametric equations for the line between $(2, 2)$ and $(2, 99)$.

   (b) Give parametric equations for the line between $(2, 2)$ and $(-99, -2)$.

   (c) How many times (and where) do the **paths** of the parametrizations $(2t, -t)$ and (t, t^2-4) intersect?

   (d) When does the parametrization $(\int_5^t \ln(x^5)\, dx, t^2 - 2t)$ come to a stop?